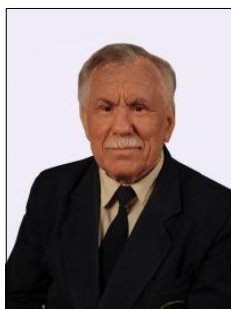


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MATHEMATICAL MODEL OF PARTICLES' MOTION IN AN AIR-FRICTION SEPARATOR

МАТЕМАТИЧЕСКАЯ МОДЕЛЬ ДВИЖЕНИЯ ЧАСТИЦ В ВОЗДУШНО-ФРИКЦИОННОМ СЕПАРАТОРЕ



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It was revealed that the separation of minerals in an air-friction separator is due to the difference in the velocities of the particles' spinning, which depend on the density, shape and size of the pieces. A mathematical model of the movement of rock particles after its descent from a curved trampoline and motion with free fall in the air is developed. Equations of motion of the particle along the horizontal and vertical coordinate axes in the horizontal airflow created by the fan are compiled. It is shown that these equations of motion can be simplified and integrated. As a result of the integration of the equations of motion, the trajectories of the motion of the rock particles are obtained after their descent from the curved trampoline and the motion under the action of gravity and the force of the air pressure coming from the fan. Based on the mathematical model of particle motion, an imitation model has been developed that takes into account the random nature of the particle density variation, the sail factor, the air velocity of the fan being fed, and the velocity of the particle's escape from the curved springboard. Trajectories of the motion of particles are shown after the withdrawal of the air-friction separator from the curved trampoline. It is shown that for the size of the rock mass + 2 ... 50 mm, the speed of air supplied by the fan should be greater than 20 m / s

Key words: *equations of motion; air speed; particle sailing; apparatuses; air-friction separator; springboard; rocks; fan; particle velocity; air environment*

Выявлено, что разделение минералов в воздушно-фрикционном сепараторе осуществляется за счет различия скоростей вращения частиц, которые зависят от плотности, формы и размеров кусков. Разработана математическая модель движения частиц горной породы после схода её с криволинейного трамплина и движения при свободном падении в воздухе. Составлены уравнения движения частицы по горизонтальной и вертикальной осям координат в горизонтальном воздушном потоке, создаваемом вентилятором. Показано, что эти уравнения движения могут быть упрощены и проинтегрированы. В результате интегрирования урав-

нений движения получены траектории движения частиц горной породы после схода их с криволинейного трамплина и движения под действием силы тяжести и силы давления воздуха, поступающего от вентилятора. На основе математической модели движения частицы разработана имитационная модель, учитывающая случайный характер изменения плотности частиц, коэффициента парусности, скорости воздуха подаваемым вентилятором, скорости схода частицы с криволинейного трамплина. Приведены траектории движения частиц после схода с криволинейного трамплина воздушно-фрикционного сепаратора. Показано, что для крупности горной массы +2...50 мм скорость воздуха, подаваемого вентилятором, должна быть больше 20 м/с

Ключевые слова: уравнения движения; скорость воздуха; парусность частиц; аппараты; воздушно-фрикционный сепаратор; трамплин; горные породы; вентилятор; скорость частицы; воздушная среда

The improvement of equipment for classification and separation of rocks is impossible without knowledge of their physical properties, which enable to determine behavior of particles in the separation zone. Finally, it will allow to develop equipment for effective separation of rocks taking into account their complex physical characteristics, in particular, frictional: static and kinetic friction coefficients, density, «sailage» [1–7]. The rocks, capable to splitting in the process of crushing, and thereby increasing the surface area acquire new properties – «sailage» for example, which may be effectively used for separation process. The separation of such minerals is carried out at the expense of the difference in their rates of spinning, depending on the density, shape and size of pieces.

Figure 1 shows the calculated scheme of a particle motion in the air flow when it leaves the curved springboard of the friction separator.

If the particle moves in a movable air medium, then the gravity force ($m\bar{g}$) and air pressure force \bar{F}_c act on it.

Taking the nonlinear dependence of the force on the speed motion of a particle (according to Newton's law), the vector force of pressure can be presented in the form

$$F_c = AC\rho_B(\dot{x}\cos\alpha + V_B)^2, \quad (1)$$

Where $A = \pi d^2/4$ is the area of the particle projection, m^2 ;

\dot{X} – particle speed vector,

d – particle diameter, m;

C – coefficient of streamlining (sailage);

ρ_B – air density (at normal temperature and atmospheric pressure, $\rho_B = 1,22 \text{ kg/m}^3$);

α – angle of the particle speed at the descent from the springboard to the OX axis, deg;

V_B – speed of air coming from the fan, m/s.

The motion of a particle in a moving air medium under the action of these forces in Cartesian coordinates is described by a system of differential equations

$$\begin{cases} m\ddot{x} = -a(\dot{x}\cos\alpha + V_B)^2 \\ m\ddot{y} = mg \end{cases} \quad (2)$$

where $a = AC\rho_B$, m is the mass of the particle, kg.

The x – axis is horizontal, y – axis is directed vertically downward, (see Figure 1).

In the first approximation, we assume that a particle has a spherical shape. The mass of the spherical particle is

$$m = \rho_x \cdot \frac{\pi d^3}{6}, \quad (3)$$

where ρ_m is the density of the material particle, kg/m^3 .

Since the rate of descent of the particle from the springboard is much less than the speed of the air supplied by the fan, a system of the equations 2 takes the form:

$$\begin{cases} \ddot{x} + k_1\dot{x} = -k_2 \\ \ddot{y} = g \end{cases}, \quad (4)$$

where:

$$\begin{cases} k_1 = \frac{3c\rho_B}{d\rho_M} V_B \cos\alpha \\ k_2 = \frac{1,5c\rho_B}{d\rho_M} V_B^2 \end{cases}$$

Integrating the first equation of the system 4, we obtain

$$\begin{cases} x = A_1 + A_2 e^{-k_1 t} - \frac{k_2}{k_1} t \\ \dot{x} = -A_2 k_1 e^{-k_1 t} - \frac{k_2}{k_1} \end{cases} \quad (5)$$

Where A_1 and A_2 are the integration constants determined from the initial conditions: $t = 0$ $\dot{x} = V_{0x} = V_H$ and $x = 0, A_1 + A_2 = 0$.

Having distinguished \dot{x} from the equation (4), taking into account the initial conditions, we obtain

$$A_1 = (V_H \cos \alpha + \frac{k_2}{k_1}) k_1^{-1} \quad (6)$$

$$A_2 = -(V_H \cos \alpha + \frac{k_2}{k_1}) k_1^{-1}. \quad (7)$$

After integration the system of the equations 4, taking into account the equations 6 and 7, we obtain the expression for the abscissa and ordinates of the particle:

$$\begin{cases} x = (\frac{V_H k_2 \cos \alpha + k_2}{k_1^2})(1 - e^{-k_1 t}) - \frac{V_B}{2 \cos \alpha} t \\ \dot{x} = (V_H \cos \alpha + \frac{k_2}{k_1}) e^{-k_1 t} - \frac{k_2}{k_1} \\ y = -V_H \sin \alpha t + \frac{g t^2}{2} \\ \dot{y} = -V_H \sin \alpha + g t \end{cases} \quad (8)$$

For simplification of calculations it is possible to determine coefficients values, enter-

ing the obtained dependences, according to the above values of the parameters: $k_2/k_1 = V_B/(2 \cos \alpha)$.

An increase of particles separation efficiency in the air may be achieved if the horizontal speed of light particles (having sailage) is «extinguished» to zero during their ascent to their maximum height and lowering to their original height at the moment of their descent from the springboard.

The time (t_0), necessary to reduce the horizontal speed to zero, may be found from the system of equations 8.

$$\dot{x} = 0 = (2V_H (\cos \alpha)^2 + V_B) \exp(-k_1 t) - V_B \quad (9)$$

$$t_0 = \ln[V_B / (2V_H (\cos \alpha)^2 + V_B)] / (-k_1). \quad (10)$$

The time of raising and lowering of particles to the level of the X axis is determined from a system of the equations 10.

$$t_n = 2V_H \sin \alpha / g. \quad (11)$$

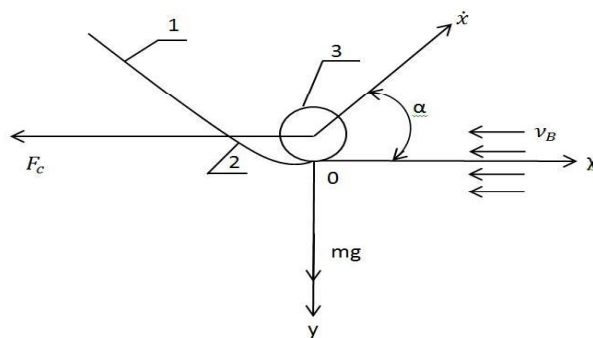
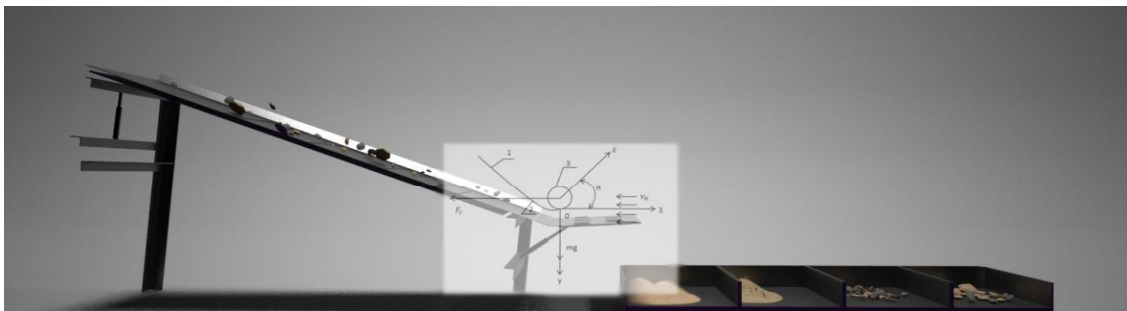


Fig. 1. Scheme of the forces' action on the particle:
1 – accelerating plane; 2 – springboard; 3 – particle / Рис. 1. Схема действия сил на частицу:
1 – разгонная плоскость; 2 – трамплин; 3 – частица

This time should be greater or equal to the time determined by the equation 10. Thus,

it is possible to find the relationship between physical and mechanical characteristics of the

rock (density, size and shape of pieces, friction coefficient determining the rate of descent of pieces from the springboard) and speed of the air supplied by the fan into the separation zone and springboard parameters.

Figures 2 - 5 show the trajectories of the rock particles, having density $\rho_m = 1000 \dots 4000 \text{ kg / m}^3$, particles size $-54 + 2\text{mm}$, coefficient of friction $0.3 \dots 0.5$. The angle of the plane inclination is 40 degrees, the springboard radius is 0,12 meters.

On the horizontal axis, each cell presents a separate container with the size (along this axis) of 0.1 m.

Figures 2-5 show that the separation of the material begins at an air speed of more than 20 m/s. At lower air speed, practically all particles will fall into one collector. Thus, by regulating the air speed, depending on the size of the rock mass and its density, it is possible to separate materials with the required efficiency.

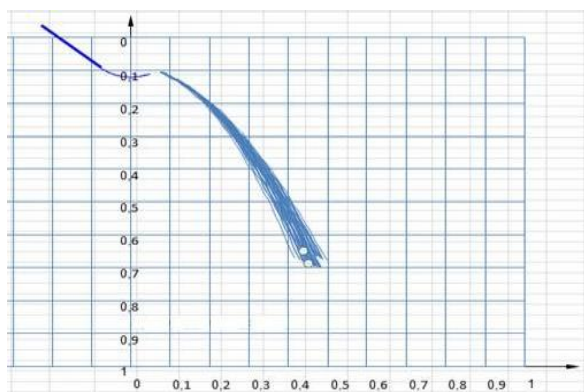


Fig. 2. Particle trajectory at an air speed of 0 m/s
Рис. 2. Траектория движения частицы при скорости воздуха равна 0 м/с/

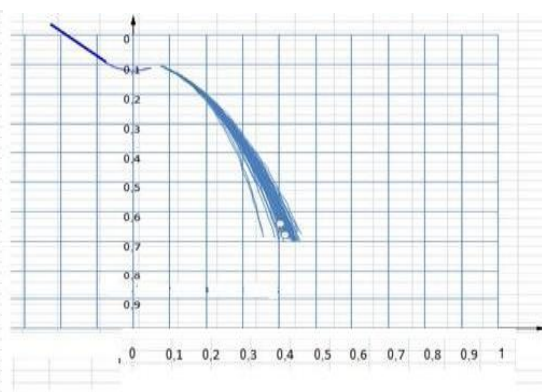


Fig. 3. Particle trajectory at an air speed of 10 m/s
Рис. 3. Траектория движения частицы при скорости воздуха равна 10 м/с

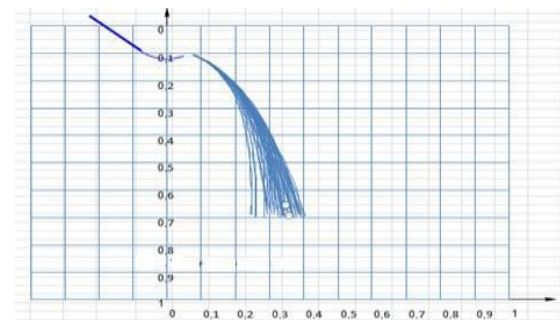


Fig. 4. Trajectory of particle motion at an air speed of 20 m/s
Рис. 4. Траектория движения частицы при скорости воздуха равна 20 м/с

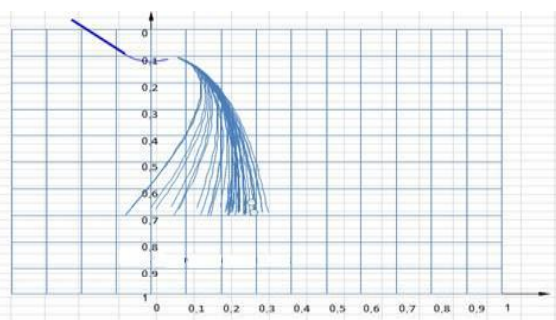


Fig. 5. Particle trajectory at an air speed of 30 m/s
Рис. 5. Траектория движения частицы при скорости воздуха равна 30 м/с

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